with the same and different isotopic spins as the ground state i.e. from states with $T = T_0$ and $T = T_0 + 1$, $T_0 + 2$. If Δ is the mean excitation energy of states in A, as defined by:

$$\sum_n |\varepsilon_{n0}|^2 = \Delta^{-2} \sum_n \langle n|V_c|0\rangle^2 ,$$

and if Δ' is the similar quantity for states with $T \neq T_0$, it is easy to show that:

$$A = \Delta^{-2}(\langle 0|V_c^2|0\rangle - \langle 0|V_c|0\rangle^2) ,$$

$$A' < [(2\,T_0 + 2)\,\Delta'^2]^{-1} \langle 0|V_c[T^2, V_c]|0\rangle .$$

We have computed the three matrix elements using Fermi gas wave-functions, ignoring space exchange, as previously described by MacDonald [1]. Using MacDonald's numerical expressions for the space integrals:

$$A = \frac{25}{36}\,(0.017\,Z + 0.37)\,\left(\frac{\Delta_c}{\Delta}\right)^2 ,$$

$$A' < 2(N - Z + 2)^{-1}\left(\frac{\Delta_c}{\Delta'}\right)^2 ,$$

where Δ_c is the Coulomb energy of a proton in the nucleus ($6Ze^2/5R$, R being the radius). There is no evident reason why Δ should be much larger than Δ_c, so we conclude that A may be ~ 1 and that states of $T = T_0$ may be strongly mixed into the ground state. The quantity Δ' must be greater than the energy needed to excite the lowest state of $T = (T_0 + 1)$, which, on the stability line, equal to Δ_c. Thus:

$$A' \ll 2(N - Z + 2)^{-1} ,$$

so that the mixing in of states with $T \neq T_0$ is very small, and isotopic spin is a quite pure quantum number.

This result, while surprising at first sight, is of very little practical significance. What it says is that the strong mixing in the self-conjugate core ($N = Z$) of a heavy nucleus is considerably diluted by the addition of the excess ($N - Z$) neutrons (which are, of course, in a pure isotopic spin state). Thus the isotopic spin purity of the whole nucleus is a rather empty result, and indicates that isotopic spin is not a very significant quantum number. This is confirmed by the fact that it is not possible to devise any experiments to really test isotopic spin purity as such in heavy nuclei. The best that can be done is to test isobar correspondence between immediately neighbouring nuclei, as has been done in the recent (p, n) studies [2,3] from Livermore. As already pointed out [4] this does not in itself imply anything directly about isotopic spin purity.

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ASYMPTOTIC PROPERTIES OF SCATTERING AND MULTIPLE PRODUCTION

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In a preceding paper [1] it has been shown that the sum over all the ladder diagrams of fig. 1 (compatible with kinematics) with just low energies cross-sections at any vertex, is an approximate solution of the strip approach to the Mandelstam representation [2] of the elastic amplitude.

In the same paper, it was suggested that the same model could be used for the evaluation of the inelastic processes. In fact, the model proposed is a simple extension to higher energies of the peripheral model [3] whose soundness in the GeV region has been recently well established [4]. The model in question, whose details shall be presented and investigated in a forthcoming publication [5], con-

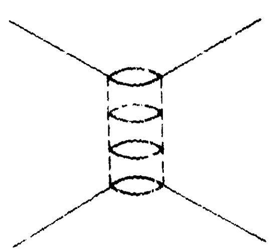

Fig. 1.

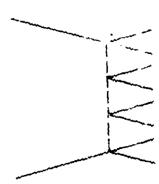

Fig. 2.

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sists in a series of elementary peripheral productions of low energy systems ("multicentre" graphs of fig. 2).

The number of those systems is limited, for every incident energy s, by phase-space and by the energy denominators given by the pion propagators. At any energy the description of the entire process shall therefore be given by a finite sum over multicentre diagrams.

The method used to sum this series is to make use of a recurrence formula in order to obtain an integral equation for the whole absorptive scattering amplitude. From the solution of this integral equation for zero transfer momentum ($t = 0$) we are able to obtain the prediction of our model for total cross-sections as well as the main properties of inelastic collisions, as spectra and multiplicity of secondaries. The solution for $t \neq 0$ allows us to investigate the diffractive elastic scattering and gives us a relativistic field theoretical way to understand the relation between asymptotic behaviours of amplitudes and existence of bound states or resonances. We shall collect in this note the main predictions of our theoretical model for the asymptotic behaviours of both inelastic and elastic scattering leaving the details and criticism to the forthcoming paper [5].

1. *Asymptotic properties of inelastic scattering*

a. Total cross-sections behave

$$\sigma(s) \propto s^{\alpha_0 - 1} , \qquad (1)$$

where s is the square of the total energy. The actual value of α_0 is given by the theory from a solution of an eigenvalue problem of a linear integral equation [6]: as a result, it is given in terms of low-energy physics (mainly an integral over low-energy $\pi\pi$ cross-section). Our little knowledge on $\pi\pi$ cross-sections does not allow for an actual evaluation: the value $\alpha_0 = 1$ (constant total cross-sections) seems to be however quite compatible with present low-energy $\pi\pi$ experimental evidence.

b. Value of total cross-sections. Fixing $\alpha_0 = 1$, the eigenfunction of the integral equation previously mentioned allows the calculation of the actual value of total cross-sections by means of a non-linear relation among amplitudes that our model provides. This calculation involves again only low-energy physics but, contrarily to the value of α_0, it is not only sensitive to integrated low-energy cross-sections, but also to the structure of it (energy of resonances). Using reasonable values of $\sigma_{\pi\pi}$ at low energy, we obtain too big high-energy cross-sections (by a factor around five or ten) as compared to experimental values. Our model indicates, however, that a more refined self-consistent calculation should be done in order to calculate the cross-sections. This refinement, whose characteristics we shall discuss later in this note, tends to remove the above-mentioned discrepancy. Contrarily to the calculation of actual numbers, the trends and energy behaviours shall not be modified by the refinements of the model.

c. Nature of secondaries. The secondaries are mainly pions. K mesons, and other particles can also be emitted with rates that can be easily evaluated for each particular case.

d. Multiplicity. The multiplicity of secondaries predicted by our model grows logarithmically with the energy.

e. Spectra of secondaries. 1. Spectra of transverse momentum. These spectra, as calculated from the theory, turn out to be independent of the incident energy as well as independent of the longitudinal momentum of the secondary itself *. The width of the spectra is characteristically a low-energy quantity. 2. Spectra of secondary energies. These turn out to be given by dE_{lab}/E_{lab} (where E_{lab} is the energy of the secondary in the lab. system) independently of the incident energy (except obviously for the fact that the maximum value of E_{lab} increases with the incident energy).

2. *Asymptotic properties of elastic amplitudes*

The absorptive part of the elastic amplitude $A(s,t)$ is given in our theory by the sum over all graphs represented by fig. 1. t stands for the square invariant transfer momentum, whose value is negative in the physical scattering region (diffractive region). The asymptotic behaviour of $A(s,t)$ as given by solving our theory is

$$A(s,t) = C(t)\, s^{\alpha(t)} , \qquad (2)$$

where $\alpha(t)$ is again given by an eigenvalue condition in a solution of a linear homogeneous integral equation [7]. Again $\alpha(t)$ is given in terms of low-energy $\pi\pi$ cross-sections. For the absolute elastic processes (no charge exchange, no spin flip) $\alpha(0) = \alpha_0$ of eq. (1). For charge exchange amplitudes $\alpha^{(0)}_{exch.} < \alpha_0$ as a result of our eigenvalue problem.

As a matter of fact [7]

$$d\alpha(t)/dt > 0 , \qquad (3)$$

and

$$\lim_{t \to -\infty} \alpha(t) = -1 .$$

Besides, the application of unitarity (in the s channel) limits

$$\alpha(t) \leq 1 \quad \text{for all} \quad t \leq 0 . \qquad (4)$$

Using fixed momentum transfer dispersion relations and using (2) one can obtain the following asympto-

* These properties seem to be in good agreement with present experimental information (see for instance G. Cocconi, 1961 meeting of Argonne Accelerator Users Group).

tic behaviour of the whole scattering amplitude $T(s, t)$

$$T(s,t) = s^{\alpha(t)}\, C(t)\, [\mathrm{cotg}\, \frac{\pi\, \alpha(t)}{2} + \mathrm{i}] \qquad (5a)$$

for symmetric amplitudes under crossing $s \rightarrow \bar{s}$ as for instance absolute elastic scattering (in which there is no exchange of quantum number between the scattering particles)

$$T(s,t) = s^{\alpha(t)}\, C(t)\, [\tan \frac{\pi\, \alpha(t)}{2} + \mathrm{i}] \qquad (5b)$$

for antisymmetric amplitudes under crossing $s \rightarrow \bar{s}$.

From eqs. (5) the elastic scattering cross-sections can easily be calculated. We find the following properties

a. For the absolute elastic processes, for which $\alpha(0) = 1$, the forward amplitude is asymptotically imaginary in the forward direction and develops a real part for increasing angle. Due to (3) the elastic cross-section decreases as $(\log s)^{-1}$; in other words, the diffraction peak is predicted to decrease logarithmically.

b. For charge exchange elastic processes ($\alpha(0) < 1$) the cross-sections decrease as $(s^{(1-\alpha(0))} \log s)^{-1}$. In this case the forward amplitudes contain a real part.

The equations obtained could well be considered also for the unphysical case $t > 0$. For $t > 4\,\mu^2$, $\alpha(t)$ would develop an imaginary part. We see from eqs. (5a) or (5b) that any time α passes through an even (odd) integer $T(s, t)$ develops a pole. We know from S matrix theory that this means the existence of a bound state for that particular value of t (or a resonance if α has also an imaginary part). Polology tells us also that the angular momentum of such a bound state is just the entire value of α in question. These poles (i.e., the relation between bound states and asymptotic properties of elastic scattering) have been found first by Regge [8] in potential Schrödinger theory and have been guessed recently to be of more general character [9]. These poles, that appear explicitly in eqs. (5), have here as a ground a field theoretical model which has the characteristics:

1. to be relativistic,
2. to satisfy, at least partially, crossing and unitary,
3. to state clearly the relation between a pole in the real part of the amplitude and the asymptotic energy dependence of both real and imaginary parts,
4. to give simultaneously the properties of inelastic scattering that seem to be in rather good agreement with experimental evidence.

It would be a legitimate question to ask whether it would be possible to obtain the condition for a bound state not through the poles of the S matrix but by a direct calculation of a relativistic bound state problem. The affirmative answer comes from the following result [6,7,8] obtained by analyzing the Bethe-Salpeter equation in the ladder approximation (represented again by the graphs of fig. 1. The condition for the existence of a bound state with total square energy t_B and angular momentum l is identical with the condition that our eigenvalue problem discussed before has a solution for $\alpha(t_B) = l$.

3. *Cuts in the angular momentum*

The simple version of the multicentre model developed up to now contains the assumption that the main contribution to total cross-sections shall be given by purely inelastic effects. Even if we know that the pure elastic (dffractive) scattering shall decrease logarithmically with the energy if compared to the inelastic one, there shall be mixed diffractive and inelastic effects, as the one represented in fig. 3, whose importance was already pointed out in peripheral calculations [10].

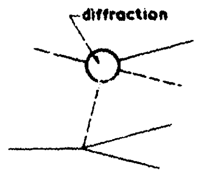

Fig. 3.

Diffraction, however, is a result of the same model, therefore a self-consistent solution of the whole elastic and inelastic problem can be attempted.

In fact, in this treatment, the elastic diffractive scattering shall not only be given by the diagrams of fig. 1 but also by those of fig. 4 in which diffraction itself appears.

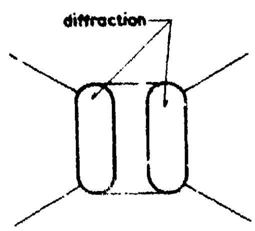

Fig. 4a.

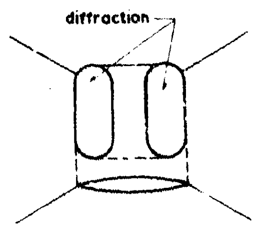

Fig. 4b.

One of the most important consequences of this more refined field theoretical approach is that it gives rise not only to the Regge poles $\alpha(t)$ previously discussed, but also to a cut (superposition of poles) *.

If we call $\alpha_c(t)$ the branch point, the asymptotic behaviour of $T(s,t)$ instead of being given by eq. (5a) and analogously for (5b) will be given by

$$T(s,t) \approx s^{\alpha(t)}\, C(t)\,[\mathrm{cotg}\,\frac{-\pi\alpha(t)}{2} + i]$$

$$+ \int_{-1}^{\alpha_c(t)} [\text{illegible}]\; s^{\alpha'}\,(\mathrm{cotg}\,\frac{-\pi\alpha'}{2} + i)\,. \qquad (6)$$

For the case $t = 0$ it is easy to see that $\alpha_c(0) = [\text{illegible}] + \alpha_0 - 1$, where α_0 is, as defined before, the [illegible] for the absorptive scattering amplitude. Because [illegible] constant high-energy cross-section!,

$$\alpha_c(0) = \alpha(0)\,,$$

i.e., the pole and the branch point coincide. The presence of such a cut will have important consequences. One of them is the impossibility (or extreme difficulty) to find experimentally any Regge pole besides the dominant one **. In fact, any secondary power behaviour s^{β}, $\beta < \alpha$, contributing to the same amplitude shall be completely masked by the integral term in (6) whose asymptotic behaviour shall be $s^{\alpha}/\log s$. Also the experimental analysis [illegible] finding out the dominant Regge pole will be complicated by the necessity of separating out the logarithmic behaviour coming from the cut contribution.

We are further investigating the theoretical identification and consequences of these cuts: our impression is that they are of much importance in the study of the analytic properties of the scattering amplitudes in the angular momentum variable.

We want to point out, finally, an already noted difficulty arising from the relation between asymptotic behaviour of cross-sections and bound states. In our field theoretical model α starts from -1 for [illegible] ** and for the no-charge or spin exchange amplitude it reaches 1 for $t = 0$, increasing continuously. This would mean that for some $t = t_G < 0$, [illegible] implying a bound state for negative square mass.

The only meaning of such a ghost is to warn physicists to disbelieve any theory up to values of t where it appears. In some papers [11] the hope was expressed that $C(t_B)$ would in some way vanish, but this is an ad hoc hope that - in our point of view - can last as long as one has not a method to calculate $C(t)$. In our theoretical model, for instance, $C(t)$ is a calculable function that does not show any chance to act as ghost killer. Our impression is that for potentials so good as to satisfy the Regge conditions - or as in our ladder theory, finite mass scalar interactions - there is no hope to avoid the ghost and simultaneously have constant total cross-sections ($\alpha(0) = 1$). This hope can perhaps be achieved by allowing for less restrictive "potentials" *; in any case this is a subject we have still under investigation whose solution, however, we need in order to trust our model, not only for calculating properties of inelastic collisions and diffraction properties, but to push it still further so as to calculate masses and properties of bound states or resonances.

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* [illegible] interesting to note that if we do a simple iteration [illegible] model by putting diagrams as those of fig. 1 at the place of diffraction in those of [illegible] we obtain the result that while diagram 1 presents only the pole, diagram 4a presents only the cut, while both pole and cut are present in diagram 4b.

** In this sense, in the framework of our theoretical approach, some speculations made recently on "Regge [illegible] analysis of high-energy events (cf. for instance [illegible], Phys. Rev. Lett. 8 (1962) 142) appear [illegible].

* The difference of our $\alpha(-\infty) = -1$ and the $-\frac{1}{2}$ limit of Regge [8] can be understood in terms of the difference of structure between the relativistic Bethe-Salpeter equation and the Schrödinger potential equation. Besides, both these results depend on the properties of the potentials for very small distances or, in our language, in the way in which high masses appear in any link of the multicentre chain of our model.

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